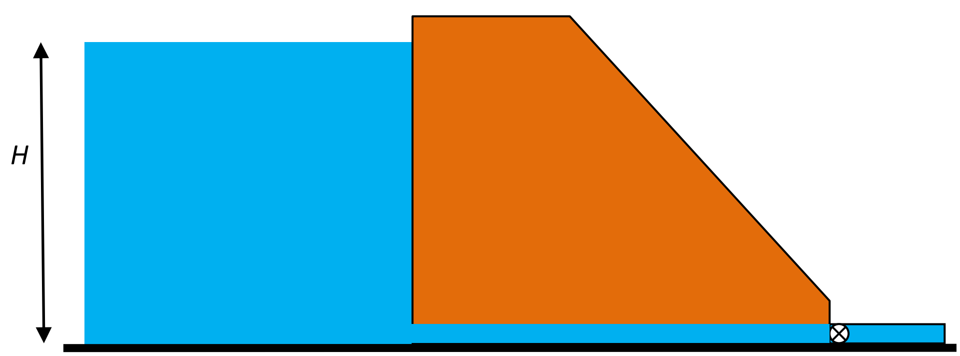
Teacher notes
Topic A

Energy density

Energy density, E_D, is defined as the amount of energy that can be extracted from a unit volume of fuel. If the volume of the fuel is V and the energy that this volume can generate is E then:

$$E_D = \frac{E}{V}$$

Pumped storage: Consider a reservoir of water of depth H.



The reservoir can be emptied through a pipe at the bottom of the reservoir. The outgoing water can turn a generator producing electricity. What is the energy density of water in this context?

The center of mass of the water is at a height of $\frac{H}{2}$ and so its gravitational potential energy is $Mg\frac{H}{2} = \rho Vg\frac{H}{2}$ where V is the volume of water and ρ is the density of water. Hence the energy density is $\frac{\rho Vg\frac{H}{2}}{V} = \frac{\rho gH}{2}$. For a height of 50 m we get an energy density of $\frac{10^3 \times 10 \times 50}{2} = 2.5 \times 10^5\ \text{J m}^{-3}$.

The energy transfers taking place here are gravitational potential energy to kinetic energy (plus losses due to frictional effects and turbulence). The kinetic energy of the water then gets transferred to rotational energy of the generator and finally electrical energy.

Uranium fission: A typical fission reaction is ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3{}^{1}_{0}\text{n}$.

The energy released can be calculated through mass differences as follows:

$$\Delta m = \underbrace{(235.0439299 + 1.008665)}_{\text{total mass of reactants}}\text{u} - \underbrace{(143.92292 + 88.91781 + 3 \times 1.008665)}_{\text{total mass of products}}\text{u}$$
$$= 0.18587\text{u}$$

Thus, for this reaction, the energy released is:
$Q = \Delta mc^2 = 0.18587 \times 1.66 \times 10^{-27} \times (3.0 \times 10^8)^2 = 2.78 \times 10^{-11}\ \text{J}$.

This energy appears as kinetic energy of the products, mostly for the produced neutrons.

This is the energy provided by the fission of a single nucleus of uranium-235 of mass approximately 235 u i.e. $235 \times 1.66 \times 10^{-27} = 3.9 \times 10^{-25}\ \text{kg}$. So 1 kg would produce $\frac{2.78 \times 10^{-11}}{3.9 \times 10^{-25}} = 7.1 \times 10^{13}\ \text{J}$. The density of uranium is $19 \times 10^3\ \text{kg m}^{-3}$ so 1 kg has volume $\frac{1}{19 \times 10^3} = 5.26 \times 10^{-5}\ \text{m}^3$. The energy density of uranium-235 is then $\frac{7.1 \times 10^{13}}{5.26 \times 10^{-5}} = 1.3 \times 10^{18}\ \text{J m}^{-3}$.

(We have calculated the energy density of pure uranium-235. U-235 has a concentration of only 0.7% of naturally occurring uranium so the energy density of uranium is different from the number calculated above.)

Gasoline: A typical value for the energy density of a fossil fuel such as gasoline is $35\ \text{GJ m}^{-3}$. So, one liter (1 L = 10^{-3} m^3) of gasoline will generate $35\ \text{GJ m}^{-3} \times 10^{-3}\ \text{m}^3 = 35 \times 10^6\ \text{J}$ of energy when burned in a car's engine. Assuming a car engine efficiency of 40% the one liter will generate $0.40 \times 35 \times 10^6 = 1.4 \times 10^7\ \text{J}$ of useful energy.

A car travelling at 120 km hr^{-1} (33.3 m s^{-1}) generates a useful power of 42 kW. One liter of gasoline will be used up in $\frac{1.4 \times 10^7}{42 \times 10^3} = 333\ \text{s}$ and in this time the car will move a distance of $333 \times 33.3 = 1.11 \times 10^4\ \text{m} \approx 11\ \text{km}$. So this car has a fuel consumption of $11\ \text{km L}^{-1}$ on the highway which is a typical value of most cars except of the very modern fuel efficient cars (this is about 26 miles per gallon). Expressed differently, to travel 1 km we need $\frac{1}{11}\ \text{L}$ so to travel 100 km we need $\frac{100}{11} \approx 9\ \text{L}$. (This is how many car magazines quote consumption: number of liters to travel 100 km.)

The power of the car is also given by $P = Fv$ where F is the force provided by the engine. At constant speed on a horizontal road, $F = F_D$ where F_D is the net drag force opposing the motion and $F_D = kv^2$ where k is a constant. Then $P = kv^3$. So, if our car is a racing car and we want to travel at 240 km hr^{-1} (at a racing circuit not a public highway!) the power will have to be increased by a factor of 2^3 = 8. So, we will need a power of $8 \times 42 = 336\ \text{kW}$ (this is way above the power of most normal cars).

One liter of gasoline will be used up in $\frac{1.4\times 10^{7}}{336\times 10^{3}}=41.2\text{ s}$ and in this time the car will move a distance of $41.2\times 66.6=2.7\times 10^{3}\text{ m}=2.7\text{ km}$. So this car has a fuel consumption of under 3 km L^{-1} .

My old SLK 350 has a power of 225 kW.